Intermittency effects inherent in turbulence

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Intermittency effects are swift variations locally seen on a time sequence of turbulent velocity. The anomalous scaling property is attributed to contamination of the inertial subrange analysis by the intermittency effects limited basically to the viscous subrange. We explain that the contamination can be removed by Gabor functions with high frequency resolutions (i.e., low time resolutions).

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The Gabor transform elucidates intermittent structures in a frequency space of turbulent velocity [1]. The Gabor transform is an analysis filter with a widely adjustable quality factor (i.e., Q factor). Let $u(t)$ be turbulent velocity, and let $g_Q(\Omega,t)$ be a Gabor function characterized by an angular frequency Ω and the Q value. The Gabor transform coefficient of $u(t)$, $G_Q[u(t)](\Omega,t')$, is defined as

$$G_Q[u(t)](\Omega,t')=\int_{-\infty}^{\infty} u(t)g_Q(\Omega,t-t')dt \ , \tag{1}$$

with

$$g_Q(\Omega,t)=\frac{1}{\sqrt{4\pi}}\frac{\Omega}{Q}\cos(\Omega t+\theta)\exp\left[-\left[\frac{\Omega}{2Q}t\right]^2\right] . \tag{2}$$

Here, G_Q denotes the Gabor transform operator at a constant Q value, and θ is a phase shift, the value of which may arbitrarily be chosen. The admissibility condition that the Gabor function must satisfy requires that $Q>2$.

The analyzing function $g_Q(\Omega,t)$ represents a Gaussian amplitude-modulated oscillation. The Gabor transform analysis extracts (angular) frequency components from the time sequence of $u(t)$. Most of the frequency components extracted are restricted within an interval $(\Omega-\Delta\Omega,\ \Omega+\Delta\Omega)$ around the centered frequency Ω, and the frequency bandwidth is defined as $\Delta\Omega=\Omega/Q$. The Gabor transform coefficient $G_Q[u(t)](\Omega,t')$ is determined from a $(4Q/\Omega)$-size part translating across the time domain of $u(t)$; therefore, $4Q/\Omega$ is the time resolution of $g_Q(\Omega,t)$. When Q is large (small), $g_Q(\Omega,t)$ is good (poor) in frequency resolution but poor (good) in time resolution.

The intermittency can be observed as scale-dependent changes in the probability density functions (PDFs) of velocity increments $u(t+\Delta t)-u(t)$ [2] and of the wavelet transform coefficients of $u(t)$ [1], and also as frequency-dependent changes in the PDFs of its electrically band-pass-filtered signals [3] and of its Gabor transform coefficients [1]. In the inertial subrange (ISR), the scale or frequency dependence of PDFs indicates the anomalous scaling property of these transformed signals. At ISR frequencies, the $2n$th order moments of $G_Q[u(t)](\Omega,t')$, $M_{2n}(\Omega,Q)$, are approximated by the anomalous scaling law

$$M_{2n}(\Omega,Q)=\langle |G_Q[u(t)](\Omega,t')|^{2n}\rangle \propto \Omega^{-\zeta_{2n}(Q)} \ , \tag{3}$$

where $\langle\ \rangle$ stands for an averaging procedure across a domain of the translation time t' and $n=1,2,\ldots$. The experimental result shown in Fig. 1, which has already been given in the previous paper [1], is that the nonlinear scaling exponent $\zeta_{2n}(Q)$ for n, i.e., the anomalous scaling property of $G_Q[u(t)](\Omega,t')$, depends on the Q value. The Q dependence of $\zeta_{2n}(Q)$ is more remarkable at larger $2n$ values. As Q gets large, $\zeta_{2n}(Q)$'s approach the Kolmogorov scaling exponents $\zeta_{2n}(Q)=2n/3$. The present study makes it clear that the Q dependence of $\zeta_{2n}(Q)$ is due to the intermittency effects inherent in turbulent velocity.

In our experiment, $u(t)$ is a streamwise component of turbulent velocity measured at a point in a jet air flow ($R_\lambda=270$). The ISR in which the scaling property is observed is from $\Omega/2\pi=0.024$ to 1.5 kHz (from 63 to 1.0 cm in wavelength) without depending on the Q value. (Our examined frequency range by the Gabor transform

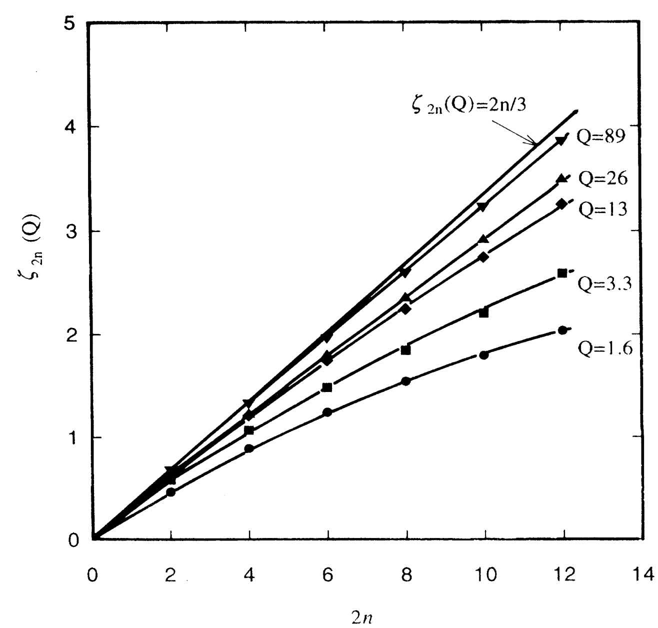

FIG. 1. Q dependence of $\zeta_{2n}(Q)$.

is from 3 Hz to 15 kHz.) The accuracy of the measurement of $\zeta_{2n}(Q)$ is from $\pm 2\%$ to $\pm 6\%$, and becomes high with an increase in Q. (The error was evaluated by the least squares method.)

We elucidate what features the intermittency effects have on the time sequence of $u(t)$ and give a lucid explanation of the Q dependence of $\zeta_{2n}(Q)$. In addition, we offer a full explanation of the following experimental results: (i) the PDFs of $G_Q[u(t)](\Omega,t')$ at frequencies in the ISR deviate more from Gaussian PDFs in larger-amplitude events (Fig. 2), and (ii) the obviously intermittent aspect of $G_Q[u(t)](\Omega,t')$ [Fig. 3(a)] is observed when Q is appropriately small and Ω is located in the viscous subrange (VSR); that is, when the analyzing Gabor function is good in time resolution.

At high frequencies in the VSR, the Gabor transform coefficients obtained at small-Q values exhibit an intermittent series of strongly amplitude-modulated oscillations, as shown in Fig. 3(a). This intermittent aspect becomes more and more notable at higher VSR frequencies. This experimental result indicates that the major factors causing the intermittent aspect exist locally in the time sequence of $u(t)$. Obviously, these factors are the intermittency effects, which should be observed as swift variations occurring in the time sequence of $u(t)$. The most striking swift variations are observed as very narrow pulselike and very short duration pulsationlike variations, some of which are indicated by arrows in Fig. 3(b). The amplitude-modulated oscillations are caused by swift variations, which an analyzing Gabor function senses.

First, let us consider a small-Q situation where the Gabor transform analysis can independently detect each of the intermittency effects. Figure 4 shows a schematic illustration of the way that the intermittency effects produce the amplitude-modulated oscillations, where the rectangular pulses in 4(a) represent the intermittency effects, the location and the strength of which are schematically represented by the width and the height of the rectangular pulses, respectively; the pulse width may be assumed to be the duration of the pulsation; 4(b) shows such a small-Q Gabor function; and 4(c) shows a time sequence of Gabor transform coefficients obtained for the pulse sequence 4(a). Thus, when Ω is located at a VSR frequency, the intermittent series of amplitude-modulated oscillations seen in Fig. 3(a) is caused by the intermittency effects inherent in $u(t)$. (It is of no importance to find a one-to-one correspondence between the amplitude-modulated oscillations and the intermittency effects.)

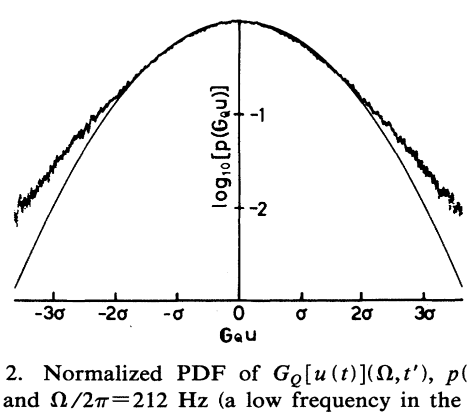

FIG. 2. Normalized PDF of $G_Q[u(t)](\Omega,t')$, $p(G_Q u)$, at $Q=3.5$ and $\Omega/2\pi=212$ Hz (a low frequency in the ISR); the solid line indicates a Gaussian curve of a standard deviation σ.

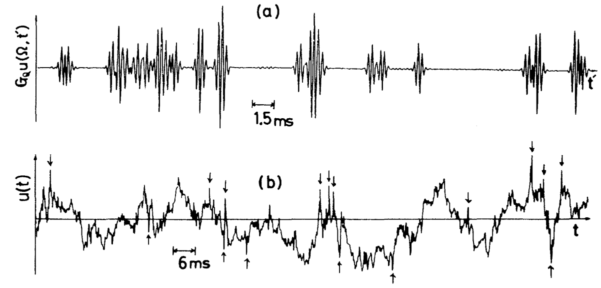

FIG. 3. (a) Time sequence of $G_Q[u(t)](\Omega,t')$ at $Q=3.7$ and $\Omega/2\pi=4$ kHz; (b) the time sequence of $u(t)$.

When Ω is larger than the ISR high-frequency end (1.5 kHz) of the turbulence used in our experiment and $Q=3.5$, the Gabor function shown in Fig. 4(b) is of a very high time resolution, such as $4Q/\Omega<1.5$ ms. At such a time resolution, the Gabor transform coefficients can hardly be affected by the main velocity components fluctuating at time scales considerably longer than 1.5 ms. Consequently, as presumed from Fig. 3(b), such a small-Q Gabor function can separate the intermittency effects of VSR time scales and the main velocity fluctuations of ISR ones rather well; therefore, when the time resolution is good enough, the Gabor transform coefficients at VSR frequencies exhibit intermittent aspects.

Next, let us consider a large-Q situation where an analyzing Gabor function can cover several or many intermittency effects: the time resolution of $g_Q(\Omega,t)$ is very poor. Then, these intermittency effects covered by the Gabor function take either the in-phase or out-of-phase mode. The in-phase effects make a positive contribution to the integral in Eq. (1), and the out-of-phase effects make a negative contribution. Hence, the effects of the in-phase and the out-of-phase factors cancel each other, so that the large-Q Gabor function decreases the contamination of the ISR analysis by the intermittency effects. When, for example, $\Omega/2\pi<1.5$ kHz and $Q=80$, the Gabor function is of a very low time resolution, such as $4Q/\Omega>34$ ms. As presumed from Fig. 3(b), such a

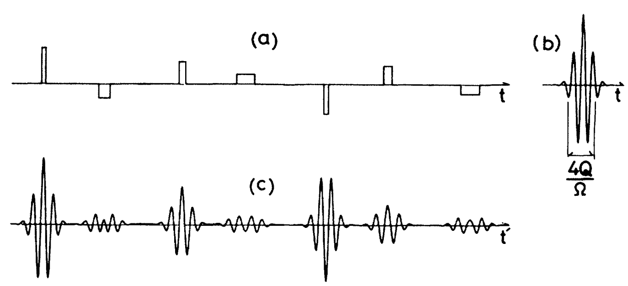

FIG. 4. Gabor transform scheme. (a) The time sequence of rectangular pulses representative of the intermittency effects; (b) the Gabor function at $Q=3.5$ (e.g., $4Q/\Omega=1.1$ ms at a VSR frequency $\Omega/2\pi=2$ kHz); (c) the sequence of Gabor transform coefficients.

large-Q Gabor function can simultaneously detect many intermittency effects. Consequently, the Gabor transform coefficients can involve slight contamination by the intermittency effects inherent in the VSR.

As the Q value increases, the contamination decreases. This decrease in contamination presents the asymptotic feature that the increase in Q makes the scaling indices $\zeta_{2n}(Q)$ close to the Kolmogorov scaling indices (Fig. 1). At poor time resolutions (i.e., large-Q values), the Gabor function cannot discriminate each of the intermittency effects. Similarly, even when Ω is located in the VSR, the large-Q Gabor functions cannot discriminate each of the intermittency effects. This fully explains the Q dependence of the PDF of the Gabor transform coefficients. The asymptotic behavior of $\zeta_{2n}(Q)$ indicates the Q dependence that the Gabor transform coefficients, with an increase in Q, take PDFs closer to Gaussian PDFs. However, when the Q value is small, the PDFs obtained at ISR frequencies have the large-amplitude events deviating from Gaussian PDFs, as shown in Fig. 2. The large-amplitude events are chiefly produced by contamination arising at small-Q values.

In conclusion, the intermittency effects are basically limited to the VSR, being swift variations in velocity fluctuations. In the limit of an infinitely long ISR, there is no anomalous scaling, i.e., no non-Gaussian intermittency. The velocity fluctuations at ISR frequencies have very weak intermittency effects, which gradually get strong at higher frequencies close to the VSR. There is an indication that the intermittency effects appear. There is some theoretical evidence that supports the VSR origin of intermittency [4].

According to the Kolmogorov theory [5], the kinetic energy cascading through an infinitely long ISR must have a constant average rate all over the frequency (wave-number) space. However, in real turbulent fluid motion, the rate of cascading kinetic energy may be assumed to fluctuate during the process of its time evolution. The fluctuations in the rate increase at smaller scales, which provide the VSR with the intermittent features. Then, the kinetic energy cascading at fast rates survives in regions of very small scales without being dissipated by the viscous effects in the process of the time evolution of turbulent fluid motion [6], and the swift variations appear locally on the time sequence of $u(t)$. The intermittency effects break down the self-similar property of turbulent velocity. The viscous effects on turbulent fluid motion are supposed to participate in the fluctuations in the energy cascade rate [6]. Hence, the intermittency effects can closely be related to the viscous effects. This reasoning does not contradict the fact that the Kolmogorov scaling law can be realized in the inviscid limit.

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